NONISOTHERMAL MODEL OF COMPRESSING A PLASTIC DISK UNDER IMPACT

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A mathematical model is considered for the thermal instability phenomenon of the axial strain of a thin homogeneous disk from an incompressible viscoplastic material with the inertial properties taken into account of the fluxes originating under mechanical impact along the axis of symmetry. These investigations are of interest for an analysis of technological material treatment processes by a pulse load, the sensitivity of explosives to impact by a pile-driver, and many other cases where the sufficiently rapid (adiabatic) compression of a thin layer of substance between rigid rough surfaces is considered.

The problem of axial compression of a non-heat-conducting disk of material with a constant shear yield point τ_S was examined in [1-4]. It follows from its solution that under developed plastic flow conditions the compressing force p is inversely proportional to the disk thickness δ, where the pressure growth continues until energy is no longer expended in the loading system. A different pattern is observed in taking account of the temperature dependence of the strength limit [5]. Because of the plastic dissipation of the mechanical energy the disk temperature T grows and as it approaches the melting point T_m (dependent on the pressure), the disk material softens and a thermal strain instability occurs, a reduction in the mean pressure as δ diminishes with a sufficient energy reserve in the loading system.

Within the framework of a noninertial pattern of substance spreading [5], it was obtained that in the limit as $T \to T_m$ the rate of pressure drop became infinite (deceleration of the impactor by the liquid interlayer was neglected). Hence, for a finer comprehension of the regularities of thermal softening of solids, the viscous and inertial properties of the disk material must be taken into account, and their influence on the magnitude of the dissipative heating of the substance must be determined, which it is especially important to know for a clarification of the reasons for the excitation of an explosion in solid high explosives under mechanical effects.

The analysis of experimental data on the impact on thin specimens of plastic substances shows [5, 6] that their compression is elastic in nature in the initial time period, where the specimen strain $\Delta\delta/\delta_0 \approx p/E$ is insignificant up to the time of reaching the substance yield point τ_S^0 on the contact surface of the impactor. The corresponding value of the specific load at the mentioned time is $p_0 = \sigma_s^0(1 + 2R/3\sqrt{3}\delta_0)$, $\sigma_s^0 = \tau_s^0\sqrt{3}$.

During the elastic stage of impact, the pressure in the specimen, and the velocity of the center of mass of the loading system vary in conformity with the formulas $p = p_X \sin \pi t/t_X$, $v = v_0 \cos \pi t/t_X$, $p_X = v_0\sqrt{MK}/\pi R^2$, $t_X = \pi\sqrt{MK^{-1}}$ obtained in [7] from an analysis of the so-called "no load" impact, i.e., in the absence of an interlayer between the impactor and the anvil. Here K is the stiffness of the loading system elements, R is the impactor radius, and E is the elastic modulus of the disk material.

There is no heating of the substance during the initial impact stage $\Delta T = 0$, and the velocity of the impactor contact surface $w = v - (\pi R^2/K)dp/dt$, $dp/dt = Ew/\delta$ is much less than v if the layer thickness is not too large.

The interlayer plastic flow starts at the time $t_1 = \sqrt{M/K} \arcsin(p_0/p_X)$ if the specimen initial thickness is greater than a certain minimal value $\delta_0 > \delta_x = (2R/3\sqrt{3})/(v_0\sqrt{MK}/\pi R^2\sigma_s^0 - 1)$. Under developed viscoplastic flow conditions, the mean pressure in the interlayer is determined from the formula [3]

$$p = \sigma_s(1 + 2R/3\sqrt{3}\delta) + 3\mu w/\delta + (\rho R^2/8\delta^2)(3w^2/2 + \delta dw/dt), \quad (1)$$

where μ is the plastic viscosity of the material, considered a constant.

Formula (1) has been obtained under the assumption $\sigma_S = \text{const}$. As is shown in [5], it is satisfied even in the case $\sigma_S = \sigma_S(T, p)$ under the condition of an equal distribution of the temperature within the disk. The power series

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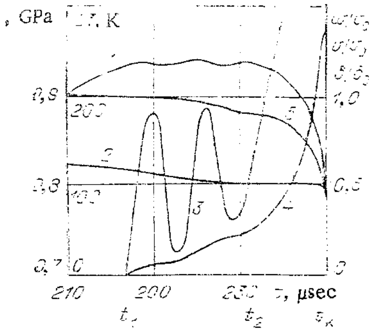

Fig. 1

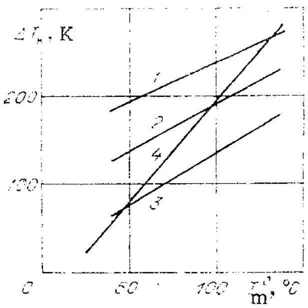

Fig. 2

$$\sigma_s = \sigma_s^0 [(T_m - T)/(T_m - T_0)]^n, \quad T_m = T_m^0 + \beta p, \tag{2}$$

is the most frequent mode of writing the function $\sigma_s = \sigma_s(T, p)$, where T_m^0 is the normal melting point of the substance.

In the case of a non-heat-conductive interlayer, or a small flow time $t \ll \delta^2 \rho c_p/\lambda$ the heat influx equation is written in the form

$$\rho c_p \frac{dT}{dt} = \frac{2\sigma_s}{3\sqrt{3}} \frac{wR}{\delta^2} + \frac{3}{2} \frac{\mu w^2 R^2}{\delta^4}. \tag{3}$$

because of the plastic and viscous dissipation of the mechanical energy. Here λ_z, ρ_s, cp are the heat conductivity, density, and specific heat of the disk material.

Together with the kinematical and dynamical relationships between δ, w, v and p

$$d\delta/dt = -w, \quad dv/dt = -p\pi R^2/M, \quad dp/dt = (K/\pi R^2)(v - w)$$

(1)-(3) form a closed autonomous system of five ordinary first-order differential equations with the initial conditions

$$\delta(t_1) = \delta_0, \quad p(t_1) = p_0, \quad T(t_1) = T_0,$$
$$v(t_1) = v_0\sqrt{1 - (p_0/p_x)^2}, \quad w(t_1) = v(t_1)/(1 + \pi R^2 E/K\delta_0).$$

The system of equations mentioned was solved numerically on an electronic computer by the Runge–Kutta method. The accuracy of the computations was 0.1-0.5%. The computation was cut off when the impactor velocity vanished or when the specimen temperature reached the melting point for a finite value of $v_k > 0$.

Results of computing the time change in the mechanical and thermal parameters are presented in Fig. 1 for impact on an elastic–plastic specimen with the characteristics: $\sigma_s^0 = 59$ MPa, E = 10 GPa, $T_m^0 = 413$°K, $T_0 = 293$°K, $\beta = 0.2$ K/MPa, n = 0.6, $\rho c_p = 2$ J/cm^3 · K, $\delta_0 = 0.13$ mm for R = 5 mm, M = 10 kg, $v_0 = 2$ m/sec and K = 200 MN/m such that $p_x = 1.14$ GPa and $t_x = 0.700$ msec. These conditions are typical for testing solid high explosives (HE) for sensitivity to mechanical actions (curve 1 is p(t), 2 is v(t), 3 is w(t), 4 is $\Delta T(t)$, and 5 is $\delta(t)$).

It is seen from Fig. 1 that the elastic stage occupies a major portion of the impact time ($t_1 = 0.217$ msec). The pressure hence grows to the quantity $p_0 = 0.93$ GPa, the velocity of the center of mass is reduced to $v_1 = 1.2$ m/sec, there is practically no motion of the impactor contact surface, and no heating of the specimen.

After passage into the plastic state, flow and heating of the specimen start, the pressure and velocity of the contact boundary increase, where the curves p(t) and w(t) are oscillatory in nature, associated with the inertial effects of the motion. As is seen from (3), the initial slope of the curve w(t) is positive for $w(t_1) \simeq 0$ and $p = p_0 + 0$, i.e., acceleration of the impactor contact surface to a velocity noticeably exceeding the velocity of the center of mass of the loading system occurs. Then w diminishes, and the whole process is later repeated. The presence of damped pressure fluctuations at the time the specimen goes over into the plastic state was observed in experiments with impact on lead disks [3].

Because of the plastic dissipation of the impact energy, the specimen is heated to 53°K, whereupon at the time $t_2 = 0.231$ msec it softens, the pressure diminishes abruptly, and the temperature grows rapidly to reach the melting point of the substance $T_k = T_m = 575$°K at the final time $t_k = 0.240$ msec. At this time the pressure is $p_k = 0.80$ GPa, the layer thickness is $\delta_k = 0.054$ mm, and the velocity is $v_k = 0.50$ m/sec while $w_k = 51$ m/sec. The maximal velocity of the radial flux of the substance ($u_k = w_k R/2\delta_k$) reaches 981 m/sec, which permits speaking about the explosive nature of the thermal softening of the specimen. Such a type of rapid rupture of thin disks from solid organic substances under impact by a pile-driver was discussed in [6].

TABLE 1

Impact parameters	Initial version	δ_0=1mm	v_0=4 m/sec	μ=10⁴ Pa·sec	β=0	K=400 MN/m	n=0	T_0=353 K
t_1, μsec	217	34	169	217	217	99	217	130
p_0, GPa	0,929	0.172	0.929	0.929	0.929	0,929	0,929	0,597
t_m μsec	27	501	6.1	9	3.8	19	143	12
p_2, GPa	0,934	0.253	0.933	0.930	0,932	0.934	1,093	0,816
t_p, μsec	8.6	45	2,6	8	2,5	8.3	0	11
p_K, GPa	0,801	0.141	0.821	0.906	0,816	0,722	1,093	0,484
T_K, K	575	435	581	598	413	559	375	511
v_K, m/sec	1,00	1.15	3,58	1,13	1.13	1,54	0	1,60
w_K, m/sec	51	7.7	101	35	75	44	0	48
δ_K, mm	0.054	0.210	0.055	0.116	0,080	0,059	0,109	0,063

Computations using a constant viscosity in (3) revealed additional information about the process. High viscosity ($\sim 10^4$ Pa·sec) results in rapid damping of the fluctuations in p and w and diminishes noticeably the appearance of other inertial effects. Moreover, it cuts down the plastic flow time $t_m = t_2 - t_1$ and the thermal softening $t_p = t_k - t_2$ but increases the value of the final temperature T_k and the layer thickness δ_k somewhat, i.e., facilitates the occurrence of an explosion.

We supplement the thermal instability pattern considered in Fig. 1 for the strain on a plastic disk by the following data. If the pressure exceeds a certain critical value $p_* = (T_m^0 - T_0)(n/\rho c_p - \beta)^{-1}$ [5], then thermal softening of the disk sets in at the time t_1 passing the plastic flow stage ($t_m \to 0$). On the other hand, if the impact energy $E_0 = Mv_0^2/2$ is insufficiently high for $p_* > p > p_0$, then plastic deformation of the interlayer proceeds without softening. In this case $t_p = 0$, and the curve p(t) grows monotonically until the time the impactor stops $v_k = 0$ by analogy with [3].

The influence of the initial conditions on the characteristic values of the mechanical and thermal impact parameters on a viscoplastic substance is seen from Table 1, where results are presented of computations for different versions of the problem considered, obtained by replacing one of the known quantities in the original version (see Fig. 1). The following deductions can be made from an analysis.

As the disk thickness δ_0 increases, the time intervals t_m and t_p grow, but the final temperature T_k diminishes. This circumstance makes excitation of an explosion difficult, as is indeed observed in experiments investigating the sensitivity of HE [8]: As the thickness increases the explosions cease in the first act of specimen rupture under impact. The law of the variation of ΔT_k due to δ_0 is written in the form $\Delta T_k \sim \delta_0^{-0.255}$ in the disk thickness range between 0.1 and 1 mm.

Practically independent of the initial impact velocity is ΔT_k, and it varies slightly with the substance viscosity, and weakly ($\sim n^{-0.164}$ for $0.4 \le n \le 1.5$) depends on the exponent n in the law (2). Let us note that if rupture occurs without thermal softening (plastic impact), then heating of the substance is reduced significantly. Thus, $\Delta T_k = 306$°K for n = 0.3, while $\Delta T_k = 160$°K (for σ_s = const $\Delta T_k = 82$°K) in the case n = 0.3 when only plastic deformation of the disk is observed for a given impact energy.

The dependence $\Delta T_k(\beta)$ is almost linear ($\sim\beta^{1.04}$) exactly as the dependence of ΔT_k on T_m^0 and σ_s^0. Graphs of the function $\Delta T_k(T_m^0)$ computed for different values (σ_s^0 = 59; 40; 20 MPa, curves 1-3, respectively) are represented in Fig. 2. Since the dependence $\sigma_s^0(T_m^0)$ is linear [8] for certain HE, the behavior of the function $\Delta T_k(T_m^0)$ with the relation $\sigma_s^0 = 0.42\ (T_m^0 - 273)$ taken into account, where σ_s^0 MPa, T_m^0 K, is shown by curve 4. In the range of values of T_m^0 between 25 and 140°C the dependence 4 is approximated by the linear function $\Delta T_k = 2.2177 T_m^0 - 31.07$.

As is seen from Table 1, a doubling of the loading system stiffness results in a certain diminution in the magnitude of the heating. This is physically conceivable since the impactor elastic strain energy $\sim\pi^2 R^4 p^2/2K$ diminishes as K increases, where part is dissipated during softening of the disk as heat going into heating of the substance.

An increase in the initial specimen temperature T_0 also diminishes T_k, which is related to the diminution in the strength σ_s^0 of the substance. These data are in good agreement with experimental results [8] on reduction of the sensitivity of HE because of preliminary heating.

Thus, a theoretical analysis has been performed of the phenomenon of thermal softening of a plastic disk under adiabatic compression conditions between colliding solid surfaces. Values of the maximal temperatures of substance heating have been computed, knowledge of which plays an important part in the analysis of the sensitivity of solid HE to mechanical effects.

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WAVE PROPAGATION IN A UNIDIRECTIONAL COMPOSITE AS COMPARED WITH A LAMINAR ELASTIC SOLID

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In studying a unidirectional composite, the assumption is often used that the bonding fibers experience only tension–compression, and the binder only shear in areas parallel to the fibers. This hypothesis is based on purely qualitative considerations, and it is apparently impossible to give a sufficiently exact a priori estimate of the error it induces. Hence, the solution of test problems and a comparison of the results obtained with the solution for an elastic laminar medium are of interest. The propagation of stationary harmonic waves along fibers and the normal incidence of a plane stress wave on a half-space are examined in this paper as such problems. When the boundary load is a Heaviside function of the time, the second problem has been considered in [1] for an approximate model. Analysis of the solution obtained showed that it possesses all the fundamental singularities inherent in even more complex problems. At the same time, consideration of a plane wave is convenient for a numerical solution since it affords the possibility of being limited to the consideration of just two adjacent layers.

1. Let the composite consist of parallel fibers of thickness h with Young's modulus E and density ρ_1 embedded in one layer, between which the spaces are filled with layers of a binder of width H and shear modulus G and density ρ_2. We take the specimen thickness as unity, direct the y axis along the boundary between the fibers and the binder, and the x axis perpendicularly to the fibers (Fig. 1). In conformity with the model taken for the composite, the equations of motion of the components in the case when all the fibers move identically are of the form [1]

$$\frac{\partial^2 u}{\partial y^2} + \frac{2G}{Eh}\frac{\partial v}{\partial x}\Big|_{x=0} = \frac{1}{c_1^2}\frac{\partial^2 u}{\partial t^2}, \tag{1.1}$$
$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{c_2^2}\frac{\partial^2 v}{\partial t^2}, \quad v|_{x=0} = v|_{x=H} = u,$$

where u and v are the displacements of the fibers and the binder, respectively, along the y axis, t is the time, $c_1 = \sqrt{E/\rho_1}$; $c_2 = \sqrt{G/\rho_2}$. The stresses are proportional to the corresponding strains $\sigma = E\partial u/dy$, $\tau = G\partial v/\partial x$.

Since the composite is an inhomogeneous body, it possess geometric dispersion which is manifest for harmonic waves as the frequency dependence of the phase velocity. Since the nonstationary waves can be repre-

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